White Paper

Volume Holographic Gratings (VHG)

Introduction

Volume holographic gratings (VHG) are diffractive elements consisting of a periodic phase or absorption perturbation throughout the entire volume of the element. When a beam of incident light satisfies the Bragg phase matching condition it is diffracted by the periodic perturbation.

Due to their high wavelength and angle sensitivity, and large information capacity and bandwidth, VHG's have been investigated intensively and developed for applications such as data storage, optical correlators, optical information encryption, fiber optic communication and spectroscopy.

Most widely used VHGs are phase gratings with low absorption. They can be fabricated in various media such as photorefractive crystals (including $LiNbO_3$ or BGO), polymers, dichromated gelatin, and photosensitive glasses. Each holographic medium has its own advantages and disadvantages with respect to photosensitivity, dynamic range (or maximum index modulation), post-exposure processing or development, material deformation, grating lifetime, and size.

Transmission and reflection gratings

Generally, VHG's can be categorized into transmission and reflection types, as shown in Fig. 1,

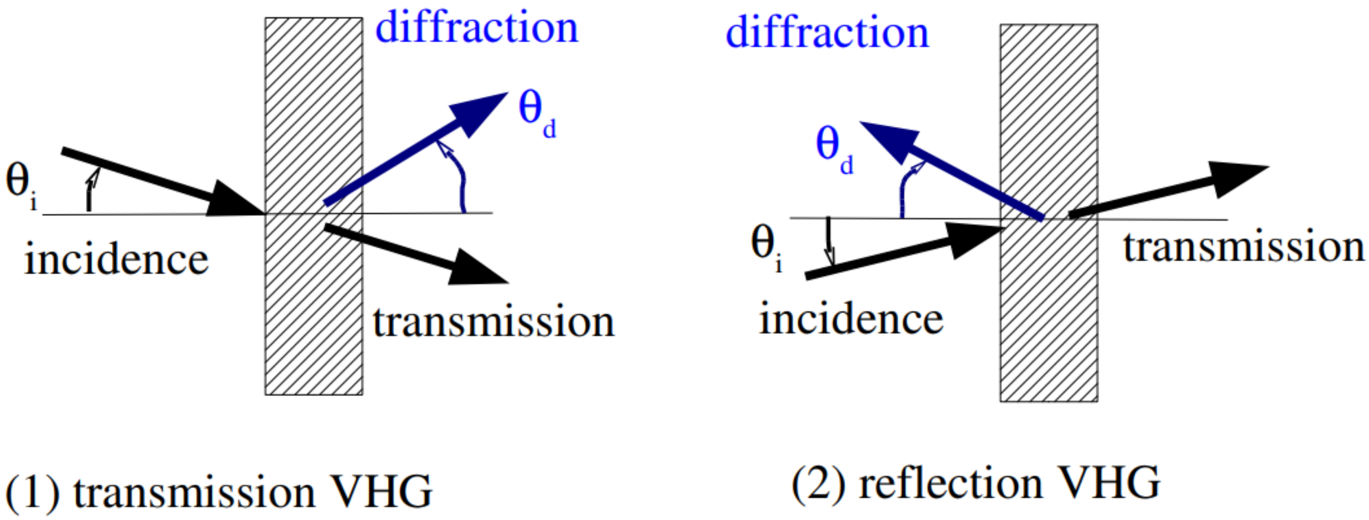

Figure 1. Two categories of VHG's: (1) transmission holographic grating; (2) reflection holographic grating. For non-slanted gratings, the magnitude of the incident angle, θ_i, and the diffracted angle, θ_d, are equal. The diffracted beam is the filtered beam.

depending on the direction of the incident and diffracted light. With transmission gratings the incident beam and the diffracted (i.e. filtered) beam are on different sides of the grating. For reflection gratings the diffracted (i.e. filtered) beam is on the same side of the grating as the incident beam.

When the orientation of the grating is not slanted relative to the surfaces, the angles of the incident and diffracted beams are symmetric with respect to the surface normal: $\theta_i = \theta_d$.

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Bragg condition and selectivity

For VHG's, the incident beam is diffracted only when the Bragg phase-matching condition is met. As shown in Fig. 2 (1), the Bragg condition of a VHG is defined as $\vec{k}_i - \vec{k}_d = \vec{G}$, where the magnitudes of the incident and diffracted wave vectors inside the medium are $|\vec{k}_i| = |\vec{k}_d| = \frac{2\pi n}{\lambda}$ with angles θ_{in}, θ_{dn} inside the medium. The grating vector magnitude is given by $|\vec{G}| = \frac{2\pi}{\Lambda}$ with grating period Λ.

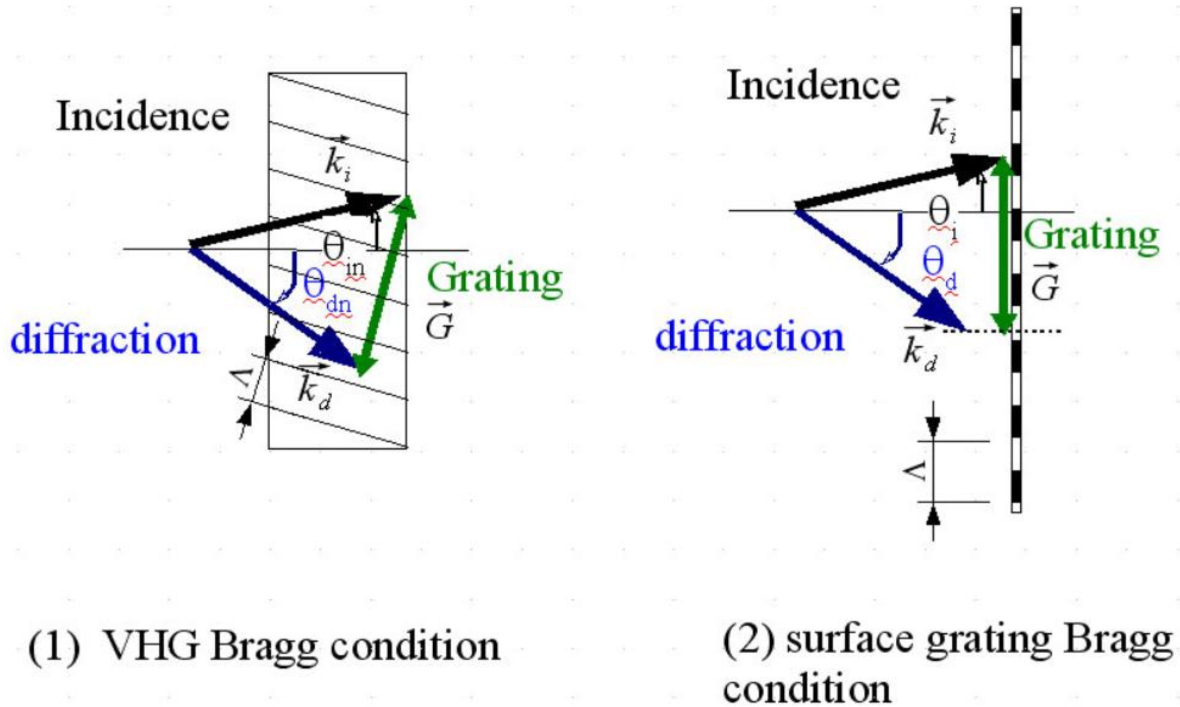


Figure 2. Bragg conditions for VHG's (1) and surface gratings (2)

By comparison, for surface gratings, Fig. 2 (2), $k_i \sin\theta_i + k_d \sin\theta_d = mG$, where the incident and diffracted wave vector magnitudes are $|\vec{k}_i| = |\vec{k}_d| = \frac{2\pi}{\lambda}$ with angles θ_i, θ_d in air, and m is any integer for the m_{th} order diffraction.

For a simple uniform phase grating, the grating is described as a periodic perturbation of the refractive index inside the medium: $n(\vec{r}) = n_0 + n_1 \sin(\vec{G} \cdot \vec{r})$, where n_0 is the average refractive index and n_1 is the grating strength as the refractive index modulation. By using coupled wave equations, the detailed diffraction properties were calculated on such uniform gratings by Kogelnik [2]. More general gratings with features such as chirping, apodization, and finite dimension can be simulated by various numerical simulation models.

Without losing generality, the simplest non-slanted transmission and reflection gratings can reveal the common characteristics of the VHG. Consider a non-slanted transmission grating with index modulation n_1 and thickness *D*. The diffraction efficiency η is defined as the ratio between the diffracted intensity and the incident intensity, without considering absorption and Fresnel reflections at the interfaces. When the Bragg condition is satisfied for wavelength λ_B, the diffraction efficiency η_B is given as $\eta_B = \sin^2\left(\frac{\pi n_1 D}{\lambda_B \cos\theta_n}\right)$, where θ_n is the incident angle inside the medium of index n. When the grating strength n_1 increases from zero, the diffraction efficiency η_B increases from 0 to 100% when $n_1 = \frac{\lambda_B \cos\theta_n}{2D}$. For even stronger gratings, the diffraction efficiency η_B will decrease from 100% because the diffracted beam is diffracted again by the same grating and coupled back into the original incident beam.

Due to the volume hologram Bragg-matching condition, VHG's have highly sensitive wavelength and angle selectivity. Considering a weak non-slanted transmission grating, i.e. $n_1 < \frac{\lambda_B \cos\theta_n}{2D}$ and $\eta_B \leq 100\%$, when the incident wavelength λ is different than the Bragg wavelength λ_B the diffraction efficiency η_λ can be approximated as a typical *sinc* function:
$\eta_\lambda = \eta_B \operatorname{sinc}^2\left(\frac{\lambda_B-\lambda}{\Delta\lambda}\right)$ where $\Delta\lambda$ is the wavelength deviation at the first null, as shown in Fig. 3 (1). Similarly, when the incident angle in air θ is different than the Bragg angle θ_B, *the diffraction efficiency* η_θ can be approximated as a typical *sinc* function:
$\eta_\theta = \eta_B \operatorname{sinc}^2\left(\frac{\theta-\theta_B}{\Delta\theta}\right)$ where $\Delta\theta$ is the incident angle deviation at the first null, as shown in Fig. 3 (2).

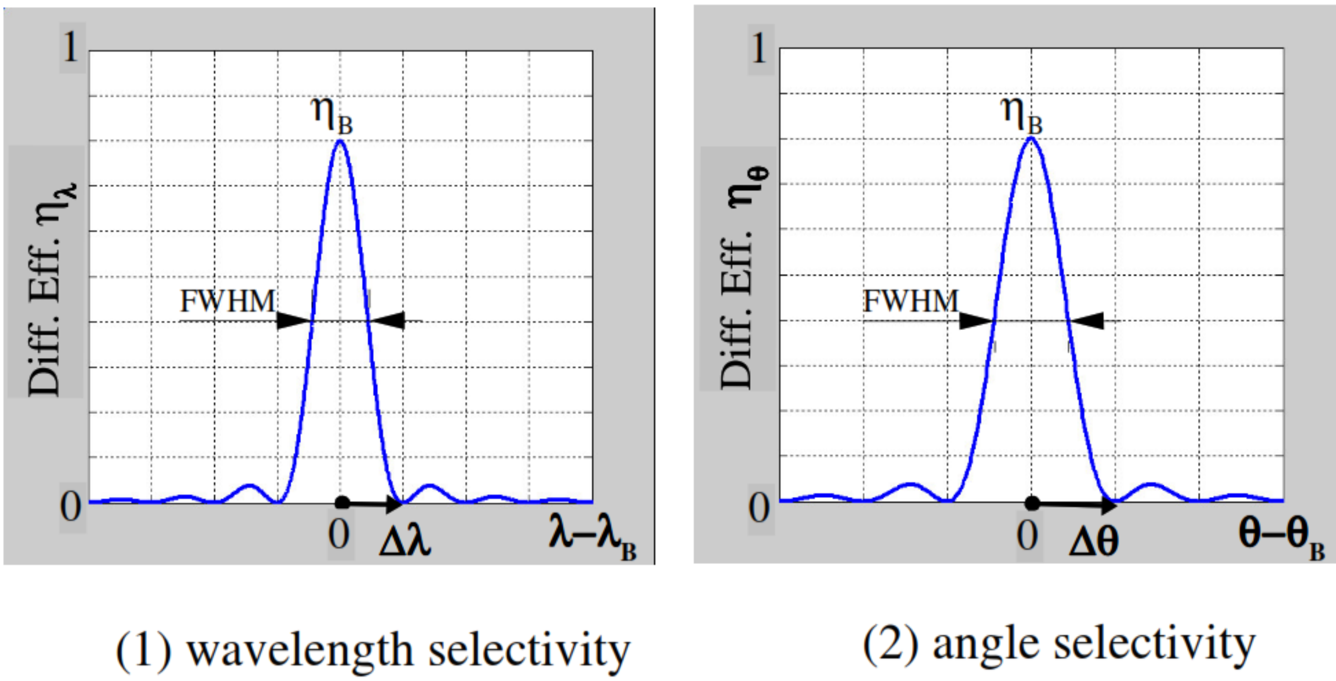


Figure 3. Typical wavelength and angle selectivity curves for VHG's. When Bragg-matched with wavelength λ_B and incident angle θ_B, the diffraction efficiency is η_B.

These *sinc* functions with respect to wavelength and angle are prototypical for all general VHG's. More detailed formulas for uniform, non-slanted transmission and reflection gratings are summarized in the attached appendices.

Comparison with surface gratings

A conventional surface diffraction grating has a periodic structure either in reflection or transmission, such as "a pattern of transparent slits (or apertures) in an opaque screen, or a collection of reflecting grooves on a substrate." [3] Compared to the surface diffraction grating, the VHG possesses a much more stringent Bragg phase-matching condition.
As shown in Fig. 2, the Bragg condition for a surface grating is $k_i sin\theta_i + k_d sin\theta_d = mG$, which could be satisfied, for the same grating G, with different diffraction order m, various wavelength $\lambda=2\pi/k$, and different incident angle θi. This leads to the existence of multiple diffraction orders and dispersion for a wide wavelength range from a single surface grating.

On the other hand, the Bragg condition for VHG is: $\overrightarrow{k_i} - \overrightarrow{k_d} = \vec{G}$, or breaking into components parallel and perpendicular to the incident surface:

$$k_i sin\theta_i + k_d sin\theta_d = G_{//}$$
$$k_i cos\theta_i - k_d cos\theta_d = G_{\wedge}.$$

This determines that for the same grating $\vec{G}$, there is only one solution of (θi, θd) for a given λ, or only one solution of λ with given θi. The diffraction occurs within a small range of wavelengths and angles around the Bragg-matching condition, as shown in Fig. 3. When the VHG thickness D→0, the wavelength and angle selectivity $\Delta\lambda$, $\Delta\theta \rightarrow \infty$ (see Appendices for details), and the VHG collapses into a tradition surface grating.

Therefore surface gratings are dispersive (i.e they diffract a range of wavelength) whereas volume gratings are not (i.e they diffract only one given wavelength)

Multiplexed gratings

Due to the limited range of wavelengths and angles over which diffraction occurs, it is possible to have multiple VHG's inside the same volume that work independently and without interfering with each other. For example, if two VHG's are recorded in the same device for two different Bragg wavelengths at the same incidence angle, the device can diffract the two selected wavelengths into different output directions with limited crosstalk.

Multiplexing capability is one of the most important and unique features of the VHG. The achievable performance and grating strength is limited by the material's dynamic range and the number of gratings multiplexed.

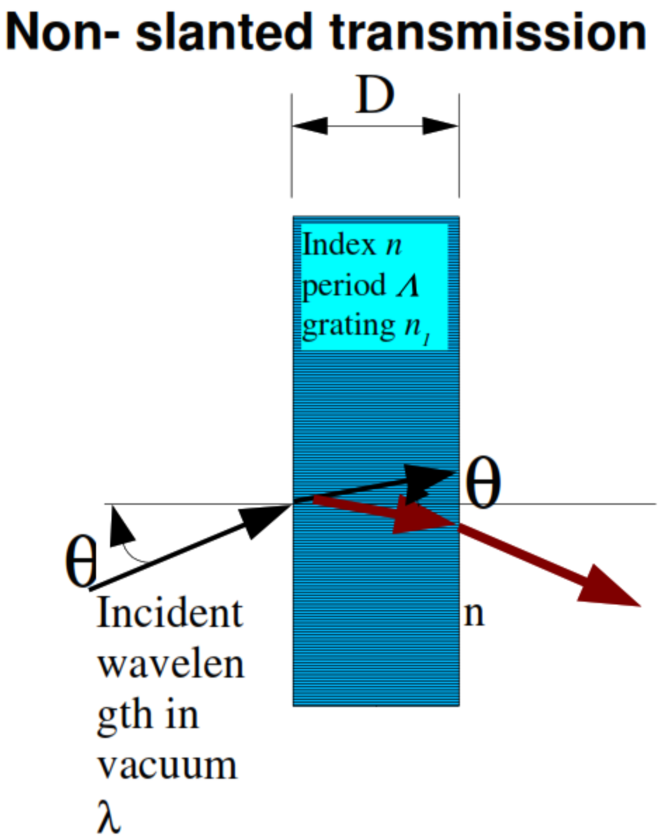
References

[1]. Introduction to Fourier Optics, J. W. Goodman, 2nd edition, pp.332, McGraw-Hill, 1996.

[2]. Coupled Wave Theory for Thick Hologram gratings, H. Kogelnik, The Bell System Tech. Journal, vol. 48, no. 9, pp. 2909-2947, Nov. 1969.

[3]. Diffraction Grating Handbook, 4th edition, pp. 9, C. Palmer, Richardson Grating Laboratory, 2000

Appendix



The formulas below provide the bandwidth of the VHG and its angular acceptance (i.e the degree of collimation required on the input beam to reach the computed bandwidth and efficiency).

Interface: $sin\theta = nsin\theta_n$

Bragg wavelength: $\lambda_B = 2\Lambda sin\theta$

Diffraction efficiency: $\eta_B = sin^2\left(\frac{\pi n_1 D}{\lambda_B cos\theta_n}\right)$

For weak gratings: $\frac{\pi n_1 D}{\lambda_B cos\theta_n} < 0.5)$

Wavelength selectivity: $\frac{\eta_\lambda}{\eta_B} = sinc^2\left(\frac{\lambda_B - \lambda}{\Delta\lambda}\right)$

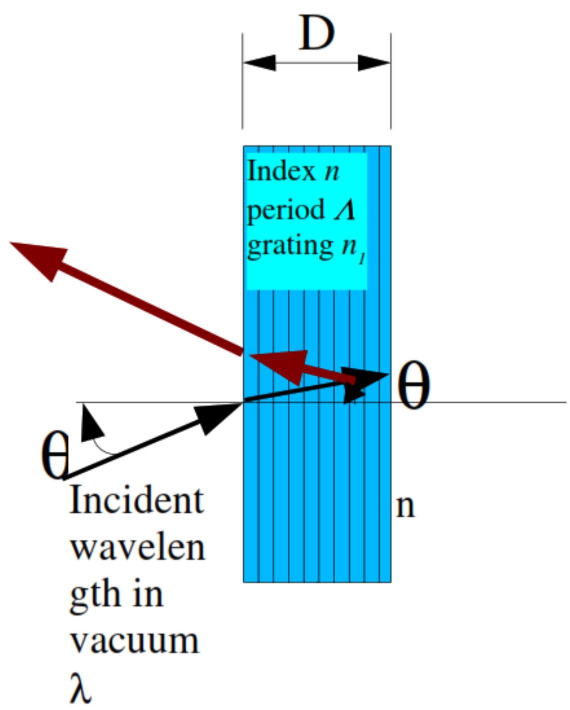
$$\Delta\lambda = \frac{\lambda^2 cos\theta_n}{2nsin^2\theta_n D}$$

$$FWHM = 0.886\Delta\lambda$$

Angle selectivity: $\frac{\eta_\theta}{\eta_B} = sinc^2\left(\frac{\theta_B - \theta}{\Delta\theta}\right)$

$$\Delta\theta = \frac{n\lambda cos\theta_n}{sin(2\theta)D}$$

Non-slanted reflection grating



The formulas below provide the bandwidth of the VHG and its angular acceptance (i.e the degree of collimation required on the input beam to reach the computed bandwidth and efficiency).

Interface: $sin\theta = nsin\theta_n$

Bragg wavelength: $\lambda_B = 2n\Lambda cos\theta_n$

Diffraction efficiency: $\eta_B = tanh^2\left(\frac{\pi n_1 D}{\lambda_B cos\theta_n}\right)$

For weak gratings: $\frac{\pi n_1 D}{\lambda_B cos\theta_n} < 0.5)$

Wavelength selectivity: $\frac{\eta_\lambda}{\eta_B} = sinc^2\left(\frac{\lambda_B - \lambda}{\Delta\lambda}\right)$

$$\Delta\lambda = \frac{\lambda^2}{2ncos\theta_n D}$$

$$FWHM = 0.886\Delta\lambda$$

Angle selectivity: $\frac{\eta_\theta}{\eta_B} = sinc^2\left(\frac{\theta_B - \theta}{\Delta\theta}\right)$

$$\Delta\theta = \frac{n\lambda cos\theta_n}{sin(2\theta)D}$$